EQUIVALENT STRESS FOR STRESS STATE WITH MOBILE PRINCIPAL DIRECTIONS IN MULTIAXIAL FATIGUE

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ABSTRACT

This paper proposes an equivalent stress with zero out-of-phase angles for the stress components with a phase difference. The equivalent stress is normalized in such a way that Crossland predictions are identical in case of in phase loadings. Therefore, we associate to the shear stress τ, defined at the generic material point M, an equivalent shear stress τ_{eq}.

Keywords: Equivalent stress, fatigue criteria, principal directions

1. INTRODUCTION

The work starts with the presentation of different types of loadings that induce the mobility of principal stress directions. Then, the effects of triaxiality, mean stress, frequency and out-of-phase loading on the mobility of principal stress directions are studied using data bases of multiaxial fatigue tests. Several methods of assessing multiaxial fatigue criteria are used in the literature. In the following, we present some of the methods widely used.

A. Bernasconi et al [1] note that a limited number of high cycle multiaxial fatigue test results have been reported in the literature for tests conducted with loads which provide both non-proportional loads and a variation of the principal stress directions with time. The only available data are those of Mielke, reported by Liu and Zenner [2], Heindenreich et al [3], Mc Diarmid [4], [5], Froustey [6], and Dietmann et al [7].

Mielke noted that the fatigue limit decreased for frequency ratios higher than $\lambda_Y = 1$ and $\lambda_{XY} = 1$. Similar results were obtained by Heindenreich et al [3].

MacDiamid results showed decreasing fatigue limits for increasing frequency ratios.

Dietmann et al. [7] reported results for biaxial sinusoidal, triangular and trapezoidal loadings, observed in all cases a decrease of the fatigue limit with respect to the in-phase loadings at the same frequency.

Multiaxial fatigue test results presented in the data bases are those conducted with loads having different frequencies are presented by Bernabes [8], Banvilet [9] and A. Bernasconi et al, [1].

The comparison between the experimental fatigue limit and the calculated ones obtained from Crossland fatigue criteria showed clearly his poor predictions capacity for stress states presenting non-zero out-of-phase angles and non-zero mean stresses .

2. TYPES OF LOADINGS AND MOBILITY OF PRINCIPAL STRESS DIRECTIONS.

There are essentially five classes of fatigue loading [10], [11]:

1. Constant amplitude, proportional loading
2. Blocks of loadings with constant amplitude
3. Constant amplitude, non-proportional loading
4. Non-constant amplitude, proportional loading
5. Non-constant amplitude, non-proportional loading

An alternative description of fatigue loadings is to talk of uniaxial stress and, multiaxial stress. As is well-known, actual loadings on the material point are often multiaxial.

On the other hand, since it is well established that failure depends on several parameters such as mobility of principal stress directions, this section is devoted to parameters that cause the change over time of principal stress directions in the fatigue loading.

2.1 Proportional loading

The loading is proportional when in a generic instant t of the cyclic history the stress state $\sigma(M,t)$, determined in material point M, is proportional to a time independent tensor $S(M)$:

$$\sigma(M,t) = \alpha(t).S(M) \text{ and } \alpha(0) = 0 \quad (1)$$

Here, α (t) is the proportionality parameter.

For the proportional load the principal stress axes do not change over time. If the principal stress axes do change, then the loading is non-proportional. In service non proportional load is most damaging. Inevitably, uniaxial load is by definition a proportional load.

It has long been recognised that changing of the principal stress directions influences fatigue phenomena. The account of mobility of principal stress directions is the object of many works in the literature [8], [12], [13], [9], [1], [14]. Many parameters are the causes of change over time of principal stress directions the fatigue loading. Among fatigue tests, these parameters we name: discontinuity of variation of principal stress directions, triaxiality, out-of-phase loading, loads with different frequency, and mean tresses.

2.2 Fatigue load frequency and mobility of principal stress directions

It is important to note that changing of the principal stress directions is not observable in all non-proportional loading. [1]. When components of the stress tensor of the loading are changing sinusoidally in time and are characterized by different frequency, the loading is considered to be non-proportional. The components of the stress tensor evolve at different frequencies or are non-correlated loadings. Table (1), collection of fatigue non-proportional experimental test with fixed and mobile principal stress directions, is a data bank of works devoted to the study of the influence loads characterized by different frequency, in structural fatigue.

2.3 Out-of-phase loading and mobility of principal stress directions

In general, the constant amplitude fatigue load stress state can be expressed as follows:

$$\sigma_{ij} = \sigma_{ija} \sin(\omega t - \varphi_{ij}) \qquad (3)$$

ϕ_{ij} in some fatigue tests will be responsible of the non-proportionality and/or the observed changing of the principal stress directions.

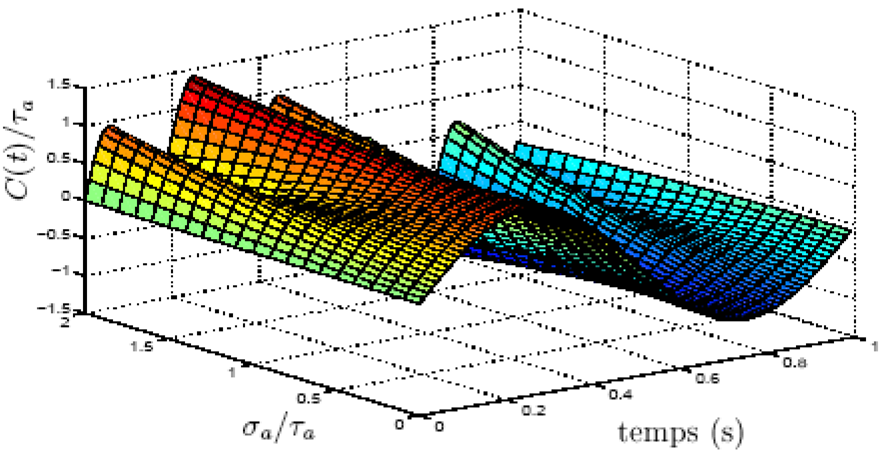

Figure 1: Temporal evolution on the maximum shear $C(t)$ of combined bending-torsion fatigue test at different frequencies $(f_\tau = 1Hz,\ f_\sigma = 5Hz)$(10).

2.4 Mean stress and mobility of principal stress directions

Non-proportionality in some fatigue loads is due to non zero-mean stress effect.

The stress tensor in Eq. (4) represents a non-proportional multiaxial stress state with fixed principal stress directions; while in Eq. (5) we have a stress state with mobile principal stress directions in time. [15].

$$\sigma(t) = \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (4)$$

$$\sigma(t) = \begin{bmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{12} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (5)$$

With: $\sigma_{11} = \sigma_m + \sigma_a \sin(\omega t)$; $\sigma_{12} = \sigma_{22} = \sigma_a \cos(\omega t)$;

2.5 Triaxiality and mobility of principal stress directions

Let's consider a specimen subjected to combine normal and torsional test. The shear stress will be responsible of the mobility of principal stress directions since principal axes rotate from their initial position.

The bellow specimen is under a torsion load superposed to the axial and radial loads a shear stress. The applied load are axial force F, the exterior pressure (P_e) and interior pressure (P_i) and the torsional couple (Mt), Fig. (2). Thus the couple induces the rotation of principal stresses (σ_1, σ_3), while the principal stress σ_2 do not vary.

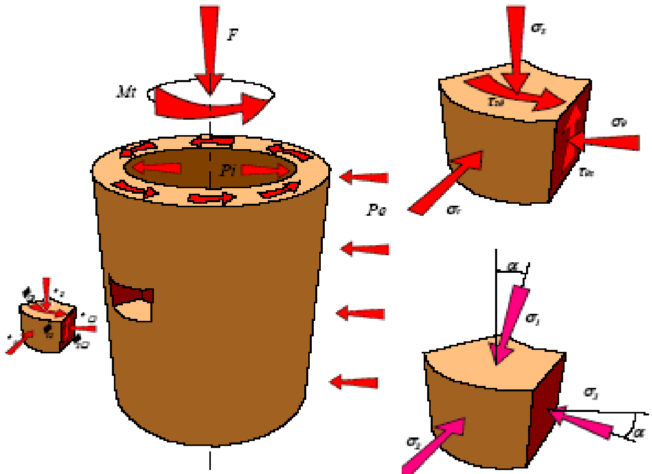

Figure 2: Stress rotation in a cylindrical sample [12]

3. BANK OF MULTIAXIAL FATIGUE TESTS CHARACTERIZED BY DIFFERENT FREQUENCY

3.1 Tests conducted by A. Bernasconi et al.

a) Test conditions

The material used for the fatigue experiments under a non-proportional loads and a variation of the principal stress directions with time is steel of grade 39NiCrMo$_3$. Monotonic tests results are summarized in table 1.

Different values of λ are 1, 2, and 3. The multiaxial fatigue tests were conducted with a constant stress amplitude ratio of $\tau_a / \sigma_a = 1/\sqrt{3}$. The frequency ratio λ=2, 3 between the shear stress and the normal stress under combined torsion and axial load are characterized by the variation of the principal stress directions with time.

Table 1: Monotonic properties of the 39NiCrMo3 steel

Young Modulus	E=206000
Tensile strength	R_m=856MPa
Tensile yield strength	$R_{0.2}$ = 625MPa

Table 2: fatigue properties of the 39NiCrMo3 steel

Type of loading	Fatigue limit
Tension-compression	$\sigma_w = 367.5 MPa$
Alternating torsion	$\tau_w = 265.0 MPa$

b) Results

Increasing λ from 1 to 2, leads to the diminution of the fatigue limit, whereas it increases when λ=3.

Table 3: Multiaxial fatigue test results

	τ_{AD} [MP]	σ_{AD} [MP]
$\lambda = 1$	294.5	170
$\lambda = 2$	259.5	150
$\lambda = 3$	266.0	153.6

3.2 Tests conducted by Banvillet

a) Test conditions

The tested material is elaborate by Renault and is used in the manufacturing of crankshafts of some engines. Combined alternating bending and torsion tests bending tests ($R_\sigma = -1$) with loads having different frequencies are illustrated in Fig. (3). Loading conditions are reported in table (5), where f_σ / f_τ is the frequency ratio between bending and torsion. The frequency of bending test is 50 Hz and 10 specimens where tested in each series.

Table 4: data bank : loads having different frequencies influence on fatigue life

Authors	**fixe principal direction axes**	**Non proportional with variable principal direction axes**	**Material**	**Conclusion**
Mielke [2]	Biaxial pulsating strress $\sigma_{xa} = \sigma_{ya}$ $R = 0.05$ $\lambda_y = 1 and 2$	Combined tension-compression and alternating torsion $\tau_{ya} = 0.5\sigma_{xa}$ $\lambda_{xy} = 1.25, 1, 2 and 8$	25 CrMo4 steel	In both cases, fatigue limit decreased for frequency ratio higher than $\lambda_y = 1$
Heindenreich et al., 1984, [3]	$\lambda_y = 2$	$\lambda_{xy} = 0.25, 1, 4$	34Cr4 steel	Similar to the first case

Mc Diarmid,1985[4], 1991[5]	Biaxial in phase or out of phase (i.e.180° out of phase) $\lambda_y = 1,2,3$	Alternating bending and torsion tests $\tau_{xya} = \sigma_{xa}$ $\lambda_{xy} = 0.25,4$	EN 24T steel	Decreasing fatigue limits for increasing frequency ratio
Froustey, 1987 [6]		Combined alternating bending and torsion tests $\tau_{xya} = \sigma_{xa}$ $\lambda_{xy} = 0.25,4$	30NCD 16 steel	Fatigue limits are pratically the same
Dietmann et al., 1991,[7]	Biaxial sinusoidal, triangula rand trapezoidal loadings $\lambda_y = 1$ and 2, and initial phase difference of 0°,90°			In all cases, a decrease of the fatigue limit with respect to the in phase at the same frequency was observed
Banvillet, 2001, [16]		Combined plane flexion and torsion tests $K_\sigma = \sigma_a / \tau_a = 1.35, 0.7$ $f_\sigma / f_\tau = 1,5$	ER7 steel	Fatigue limit decreases when $K_\sigma > 1$
Benabes, 2006, [8]		Combined plane flexion and torsion tests $K_\sigma = \sigma_a / \tau_a = 1.61, 0.61$ $f_\sigma / f_\tau = 1/8, 8$	***cast iron*** EN-GJS800-2	Fatigue limit decreases when $K_\sigma < 1$
A Bernasconi et al, 2005, [1]		Combined compression and torsion tests $\tau_a / \sigma_a = 1/\sqrt{3}$ $\lambda = f_\tau / f_\sigma = 1, 2,3$	39NiCrMo $_3$ steel	reduction of the allowable stress amplitude for λ = 2, whereas the subsequent increase for λ = 3 is negligible for practical purposes

b) Results and analysis

Data reported by Froustey [6], refer to combined alternating bending and torsion tests on plain specimens at 10^6 cycles on the 30NCD 16 steel with frequency ratios $f_\sigma / f_\tau = 4$ and

$f_\sigma / f_\tau = 1/4$; a constant stress amplitude ratio $K_\sigma = \sigma_a / \tau_a$ showed that the fatigue limits are practically the same irrespective of which is the stress component of higher frequency.

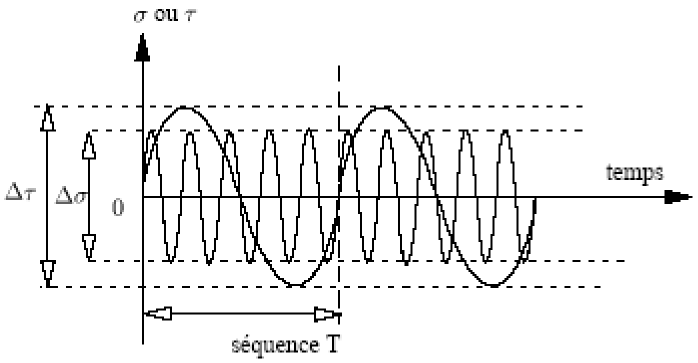

Figure 3: combined alternating bending and torsion tests at different frequencies [9].

Tests realized on cast iron EN-GJS800-2 are not in contradiction with results obtained by Froustey [6]. The results show the influence of stress amplitude ratio $K_\sigma = \sigma_a / \tau_a$ on the fatigue life for combined alternating bending and torsion tests at different frequencies; the influence is all the more important as K_σ is grater than one.

Fig. 4 shows the influence of $K_\sigma = \sigma_a / \tau_a$ on the fatigue life on tests at different frequencies.

Table 5: load conditions and results of sinusoidal combined alternating bending and torsion tests at the same and different frequencies

σ_a	τ_a	f_σ / f_τ	K_σ	$N_{r50\%}$
225	167	1	1.35	122 191
225	167	5	1.35	49 645
155	204	1	0.76	119 911
155	204	5	0.76	86 785

Table 6: Monotonic properties of cast iron EN-GJS800-2

Material	**E** (MPa)	ν	R_m (MPa)	$R_{e0.2}$ (MPa)
Cast iron EN-GJS800-2	164900	0.275	795	462

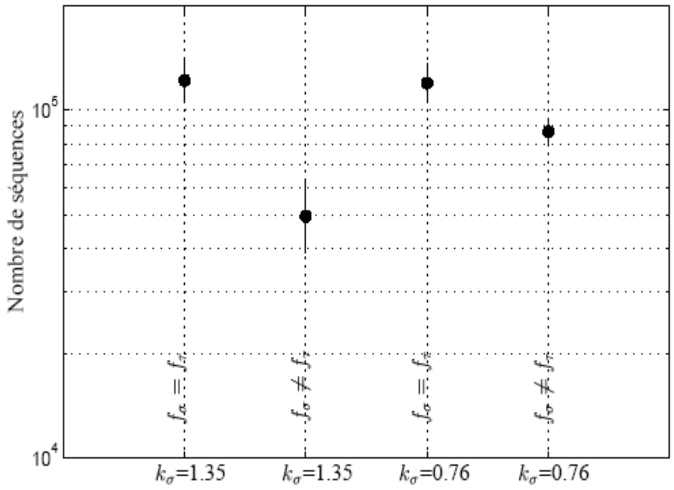

Figure 4: Results on cast iron EN-GJS800-2, of sinusoidal combined alternating bending and torsion tests at the same and different frequencies

Table 7: Fatigue limits at 10^6 cycles on cast iron EN-GJS800-2 plain specimens

Material	$\sigma^D_{Trac-1}(MPa)$	$\sigma^D_{Fp-cyl}(MPa)$	$\sigma^D_{Fr-1}(MPa)$	$\tau^D_{-1}(MPa)$
Cast iron EN-GJS800-2	245	294	280	220

3.3 Tests conducted by Benabes: combined alternating bending and torsion tests with phase shift $(\phi = 90)$ at different frequencies

The tested material is steel ER7; monotonic and cyclic properties, load conditions are given in tables 9 and 10. f_σ / f_τ is the frequency ratios between bending and torsion ; The frequency of bending test is 48 Hz and 8 specimens where tested in each series.

The results show the influence of stress amplitude ratio on the fatigue life for tests at different frequencies; when K_σ is grater than one, the influence of $K_f = f_\sigma / f_\tau$ on the fatigue life is negligible; when $(K_\sigma < 1)$ the influence of $K_f = f_\sigma / f_\tau$ on the fatigue life is pronounced.

Table 8: load conditions and results of sinusoidal combined alternating bending and torsion tests at different frequencies.

$\sigma_D / \sigma^D_{-1}, trac$	f_σ / f_τ	K_σ	$N_{r50\%}$	$\tau_a / \sigma^D_{-1,trac}$
1.08	8	1.61	11036	0.67
1.08	1/8	1.61	10695	0.67

0.5	8	0.61	34793	0.8
0.5	1/8	0.61	11529	0.8

Banvilet [9] through tests on cast iron EN-GJ800-2 showed the influence of $K_\sigma = \sigma_a / \tau_a$ on fatigue life. The influence is as important as K_σ is grater than one.

Table 9: Monotonic properties of steel ER7.

Material	$R_{e0.2}\,(MPa)$	$R_m\,(MPa)$
ER7	400	678

Table 10: fatigue limits of steel ER7 at 2.10^6 Cycles

Material	$\sigma^D_{Fp-1}\,(MPa)$	$\sigma^D_{Fr-1}\,(MPa)$	$\tau^D_{-1}\,(MPa)$
ER7	296	284	198

Fig. 5 shows the non proportionality due to different frequencies, through the variations of principal σ_{II} stress with principal stress σ_I .

4. METHODS OF ASSESSMENT

Several methods of assessing multiaxial fatigue criteria are used in the literature. In the following, we present some of the methods widely used in the literature.

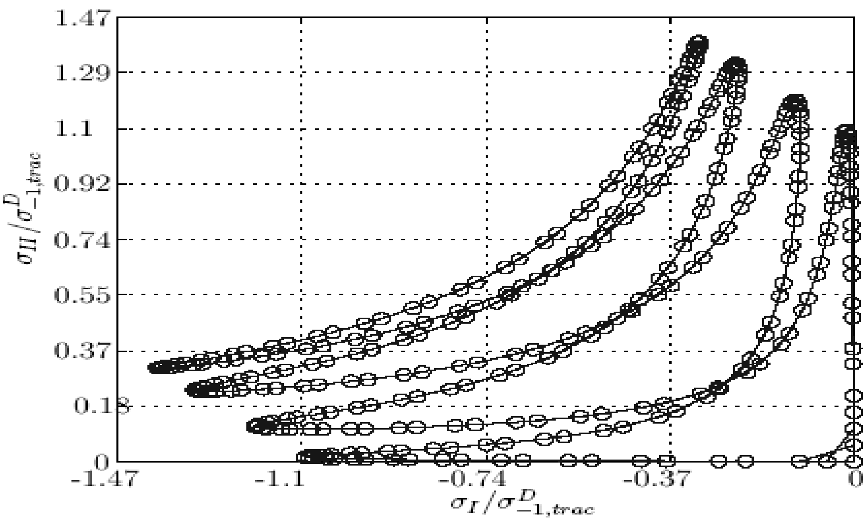

Figure 5 : Evolution de la contrainte principale σ_{II} en fonction de la contrainte principale σ_I lors d'un essai sous chargement de flexion plane et de torsion combinées $(R_\sigma = -1)$ à fréquences différentes $f_\sigma = 8Hz,\ et\ \sigma_a = 1.08\sigma^D_{-1,trac}$ [8]

4.1 Use of a factor of safety

In a multiaxial state of stress, the fatigue behaviour is very complicated. A multiaxial fatigue criterion is generally an instrument that permits the determination of the fatigue limit through an equivalent uniaxial stress with zero-mean stress. The accuracy of the criterion is measured through the coincidence of theoritical fatigue limit and the experimental fatigue limit over the same number of load cycles or fatigue life. To assess fatigue criteria, some authors define a factor of safety C_s. In the plane (X_1, X_2), the domain of no crack initiation given by the criterion can be defined by a straight line segment [8]. The equation of this straight line is mostly defined by the experimental value of two simple fatigue tests. The in service stress state at the material point P, has an image, point M, in the space constituted by (X_1, X_2), of the criterion. If point M is in the safety domain, Fig. (6.a), the structure will support the solicitation; the safety factor is then define as: $C_s = \frac{ON}{OM} > 1$. Here N is the intersection of line OM with the fatigue limit domain assumed as the straight line with a constant slope. If point M is in the crack initiation domain : $C_s < 1$ Fig. (6.b), the structure will not support the solicitation. From the value of C_s and depending on testing results and previsions, we can have three possibilities:

- When $C_s = 1$, the criterion yields exact previsions;
- When $C_s > 1$, the criterion yields non conservative previsions;
- When $C_s < 1$, prévisions are conservative.

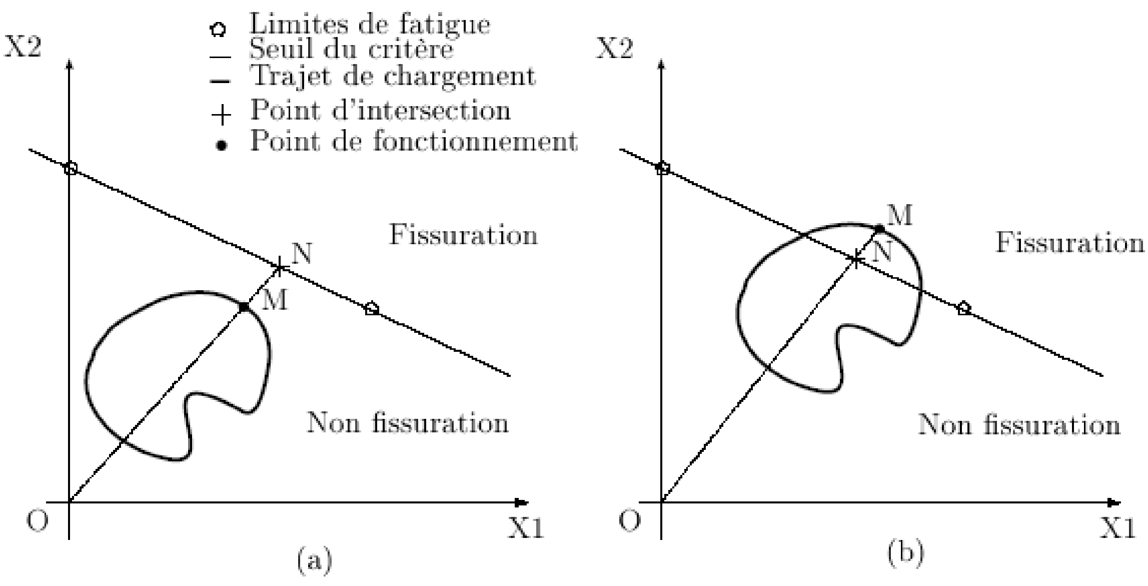

Figure 6: Principle of plane representation of two variables criterion (X_1, X_2) [8].

4.2 Use of relative error

Another tool in assessing fatigue criteria or comparing experimental fatigue limit data with the model predictions is the definition of a fatigue relative error. Relative error as defined in Eq. (3) is obtained from the experimental fatigue limit $\sigma_{a,\exp}^{D}$ and theoretical fatigue limit given by each criterion.

$$ERP(\%) = \frac{\sigma_{a,\exp}^{D} - \sigma_{a,calcul}^{D}}{\sigma_{a,\exp}^{D}} \quad (3)$$

For a multiaxial load, the relative error is evaluate when the mean stress, the load ratio $K_{\sigma} = \sigma_a / \tau_a$ are fixed.

- If $ERP \geq 0$, previsions are said to be conservatives.
- If $ERP < 0$, previsions are non conservatives

4.3 Use of fatigue strength

Redlining the fatigue criteria through the fatigue strength lead to the assessment of fatigue criteria; that is to locate theoretical fatigue limit to experimental fatigue limit at N cycles reported in the literature.

According to Weber [13], normalisation of a fatigue criterion consists in defining fatigue strength (E) of the criterion. When the fatigue limit is attained, the fatigue strength yields the value one, for a constant amplitude multiaxial fatigue load. The fatigue strength helps to locate the multiaxial fatigue load with respect to the fatigue limit at N cycles. The meaning and interpretation of fatigue strength error index is as follows:

- When $E < 1$, fatigue limits estimations of the different multiaxial fatigue criteria are non conservative;
- When $E > 1$, predictions are conservative;
- When $E = 1$, fatigue limits estimations of the different multiaxial fatigue criteria are exact.

The accuracy in estimating fatigue limits of the different multiaxial fatigue criteria have been quantified through the fatigue strength error index ΔI. Expressed in percentage $\Delta I(\%)$, the fatigue strength error index is defined as:

$$\Delta I(\%) = (E - 1) \quad (4)$$

When the value of ΔI is close to zero, fatigue limits predictions are exact. If ΔI is positive predictions are conservative, while for a negative ΔI, predictions are said to be non conservatives. **It is possible to evaluate the fraction of the total number of experimental fatigue tests for which satisfactory predictions are given by a fatigue criterion;** by evaluating the fatigue strength error index for both stress states with fixed and mobile principal stress directions.

4.4 Analogy to solid mechanics failure criteria

Some authors choose to compare the equivalent uniaxial stress defined by the criterion to some fatigue characteristics of the material. A given point in the body is considered safe as long as the equivalent uniaxial stress is at that point is under the fatigue limit; however damage is possible when the equivalent uniaxial stress at a point in the body exceeds the fatigue limit. Mathematically, this is expressed as,

- $R_{crit} \leq 0$, no damage
- $R_{crit} > 0$, damage is possible

Where R_{crit} is the expression for the criterion.

4.5 Presentation of some fatigue criteria

Analysis of results obtained by Weber [13] on 37 fatigue criteria permit to distinguish Dang Van criterion and Fogue criterion. The previous criteria yielded good predictions from loads of the data bank made up of 233 experimental fatigue results, for stress states presenting non-zero out-of-phase, non-zero mean stresses, biaxiality. Some of the criteria like those proposed by Crossland, Papadopoulos have the tendency to yield non conservative predictions.

The description of the fatigue loading resulting in a multiaxial stress state at a material point is done through sinusoidal components $\sigma_{ij}(t)$. In a generic instant t of the cyclic load history is:

$$\sigma_{ij}(t) = \sigma_{ijm} + \sigma_{ija} \sin(\omega t - \phi_{ij}) \quad (5)$$

From testing on various fatigue loadings, Weber acknowledged that none of the 37 fatigue criteria yielded perfect predictions over all the types of fatigue experiments.

Banvillet [9] justified when comparing his model to Dang-Van, Papadopoulos, and Crossland criteria by the fact that these approaches are well known and are widely used in industries.

Delahay [16] assessed the prediction quality of the following multiaxial fatigue criteria: Dang-Van, Papadopoulos, Crossland, Morel and LAMEFIP; by confronting theoretical results to fatigue experimental results obtain on plain specimens (without notches). LAMEFIP and Morel multiaxial fatigue criteria rendered the best results for proportional and non- proportional load histories. The other criteria such as Dang-Van, Papadopoulos, and Crossland yielded quite poor predictions with error index out of an error interval of $\pm$10%; thus from theoretical considerations it appears that those criteria do not include the influence of changes of the principal stress directions under a multiaxial loading on the material fatigue. Furthermore the results are non conservative (unsafe), thus letting their utilisation by design and manufacturing engineers to be dangerous. The comparison between the experimental fatigue limit and the calculated ones obtained by Dang Van and Crossland fatigue criteria showed clearly their poor predictions capacity for stress states presenting non-zero out-of-phase angles and non-zero mean stresses .

In the work of B. Kenmeugne [14] and B. Soh Fotsing [17], evaluation of the predictive capabilities of Robert criterion and Fogue criterion under multiaxial fatigue loading with fixed and mobile principal stress directions is done. The author justifies the capacity of global approaches fatigue criteria like Fogue criterion to account for the mobility of principal stress directions by the fact that in this type of loads, a set of planes are critical. On the other hand, when principal stress directions are fixed, the plane sustaining the greatest stress (maximum shear stress), the most damaged one remain the same.

In the following, the application of global approaches has been restricted to a famous criterion, namely Crossland criterion, based on a linear combination of the second invariant of the stress deviator, which is as a measure of the averaged shear solicitation to fatigue along the loading history over all the planes passing through a given material point; and the maximum hydrostatic pressure which is the quantity obtained by averaging the normal stress over all the planes passing through a given material point. Also, a modification of Crossland criterion will be proposed for better predictions of experimental data when multiaxial stress states present

5. INTRODUCING THE MOBILITY OF PRINCIPAL STRESS DIRECTIONS

We propose a new procedure for the evaluation of parameters appearing Crossland criterion in such a way that the poor predictions, as reported in table 11 should be more conservative; with a fatigue strength error index close to zero.

Thus, a new procedure is proposed by defining an equivalent stress with zero out-of-phase angles for the stress components with a phase difference. The equivalent stress is normalized in such a way that Crossland predictions are identical in case of in phase loadings. Therefore, we associate to the shear stress τ, defined at the generic material point M an equivalent shear stress τ_{eq} defined such as:

$$\tau = \tau_m + \tau_a \sin(\omega t - \varphi) \tag{6}$$

$$\tau_{eq} = \tau_m + \tau_a \left(|\cos\beta + \sin\beta|\right)^n \sin(\omega t) \tag{7}$$

With n a real and

$$\beta = (\delta_{0\varphi} - 1 + \varphi) \tag{8}$$

Nota: δ is Kronecker symbol

Later on, Crossland* criterion shall represent Crossland criterion when applied using the above equivalent stress.

Table 11: Results with quite poor predictions

Tests	2-3	2-9	2-11	3-2	3-6	3-9	4-2	4-6	4-9
$\lvert\Delta I\rvert * (-1)$	22.93	23.7	25.5	28.14	28.89	23.99	27.27	25.12	14.97

5.1 General study

The study of the influence of parameter n in predicting fatigue strength error index trough Crossland* criterion is done for the following values of parameter $n: 1/2, 1/4, 1/8, 1/16, 1/32$.

The predictions of fatigue strength provided by Crossland and Crossland* for several materials under in-phase and out-of-phase alternated bending and torsion loading are reported by K. M. Tsapi [18], and tables 12, 13, 14, 15 and 16.

5.2 Analysis of results

The quality of the predictions deduced by the criteria examined can be evaluated through fatigue strength error index, ΔI. The absolute value of the error index ($|\Delta I|$) is defined and the number of experimental tests whose error index falls in each interval of 5% range is considered with respect to the values of parameter n and the total number of tests. The results of such an analysis are shown in tables 12, 13, 14, 15 and 16.

Table 12: Predictions for loadings with fixed principal stress directions (20 tests)

$\|\Delta I\|$	**Crossland**	**Crossland***
5%	80%	80%
10%	100%	100%

The results for loadings with mobile principal stress directions are reported in the tables bellow with respect to some causes of mobility of principal stress directions like non-zero out-of-phase angles, non-zero mean stresses or the combine effect of non-zero mean stresses and non-zero out-of-phase angles.

Table 13: Crossland* predictions for stresses with non-zero out-of-phase (22 tests)

$\|\Delta I\|$	**Crossland**	**Values of parameter n in Crossland* criterion**				
		1/2	**1/4**	**1/8**	**1/16**	**1/32**
5%	40.91%	27.27%	31.82%	54.55%	54.55%	63.64%
10%	63.64%	59.09%	95.45%	100%	100%	100%
15%	81.82%	90.91%	100%			

Table 14: predictions for non-zero mean stresses (7 tests)

$\lvert\Delta I\rvert$	**Crossland**	**Crossland***
5%	71.43%	71.43%
10%	85.71%	85.71%

Table 15: Crossland* predictions for combine effect of non-zero mean stresses and non-zero out-of-phase (12 tests)

$\lvert\Delta I\rvert$	**Crossland**	**Values of parameter n in Crossland* criterion**				
		1/2	**1/4**	**1/8**	**1/16**	**1/32**
5%	8.33%	41.67%	50%	58.33%	66.67%	66.67%
10%	25%	91.67%	100%	100%	100%	100%
15%	58.33%	100%				

Tables 16 give recapitulative of predictions for loadings with mobility of principal stress directions at any material point M, assumed to be the critical one for the component integrity, using Crossland criterion, the proposed equivalent stress in predictions with Crossland criterion (Crossland* criterion). The tables revealed the criteria that show, in general, satisfactory predictions of fatigue strength error index, regardless of the mobility of principal stress directions.

Table 16: Recapitulative of Crossland* predictions for loadings with mobile principal stress directions (41 tests)

$\lvert\Delta I\rvert$	**Crossland**	**Values of parameter n in Crossland* criterion**				
		1/2	**1/4**	**1/8**	**1/16**	**1/32**
5%	36.59%	39.02%	43.58%	58.54%	60.98%	65.85%
10%	56.10%	73.17%	95.12%	97.56%	97.56%	97.56%
15%	76.61%	92.68%	97.56%			

6. CONCLUSION

From the recapitulative, we can make the following conclusions:

- The influence of parameter n is meaningful for loadings with mobile principal stress directions, especially when the cause of mobility of principal stress directions is non-zero out-of-phase angles.
- The closest value to zero of fatigue strength error index for both mobile and fixed principal stress directions are obtained when $n = 1/32$; independently of the value of parameter n , the re-formulated Crossland criterion always provided best predictions estimates than all the other criteria considered in the present study.
- Dependent upon the value of parameter n, application of the proposed equivalent stress leaded Crossland criterion to yield good predictions whenever the principal stress directions are mobile.
- Analysis of results presented by K. M. Tsapi [18], shows that parameter n should be correlated to the material of study.
- We think that optimal predictions should be obtained by defining for each material, parameter n which yields closest results to experimental data.
- For stress states with mobile principal stress directions, Crossland* criterion yielded better predictions estimates than Crossland criterion.

ACKNOWLEDGEMENTS

The authors thank the French cooperation in Cameroon, the Alsatian agency for international development IRCOD and the Alsatian universities of technology for their support.

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